

# Optimal Allocation of Rural Energy Resources Using Goal Programming -A Case Study

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**Abstract:** The optimal allocation of rural energy resources to various energy end users is an important aspect for bridging the energy supply and demand gap in India. It has been observed that allocation should be channelized by multiple criteria. In this paper a multi objective programming model for such an allocation process is presented. A standard model has been applied for rural sector of Malnad region of Shimoga District, Karnataka state. The data used for the analysis is obtained from detailed survey of various energy resources and demands of Hosur Village. The optimization of energy resources has been done by Goal Programming method. The integrated energy plan for selected village is prepared for 100% coverage basis, covering geographic and demographic characteristics, economic activities and general living condition of the people and present energy consumption pattern. Data is used for optimization of rural energy sources using goal programming taking three objectives into account i.e. minimization of cost, maximization of use of locally available resources and minimization of use of external energy sources. For achieving this, linear programming model is prepared initially and then its feasibility is checked. Since it is a multi-objective programming, QM (Quantitative Analysis for Management) for Windows has been used for solving the goal programming. The goal programming model is prepared and results obtained have been compared with the present energy consumption pattern. The results of the model indicate that the use of agro waste and firewood should be promoted for heating and cooking respectively.

**Index Terms:** Rural energy resource allocation, multiple objectives, Goal programming, Sustainable development.

## I. INTRODUCTION

India, being an agricultural country which has large resources of agro residue, if it is utilized efficiently can solve the problem of energy crisis to a large extent. Energy is universally recognized as one of the most significant inputs for economic growth and human development. There is a wide gap between energy supply and demand. One important strategy towards bridging the gap is to optimally allocate various energy resources to the energy needs arising out of wide variety of end-uses for a given geographical region. The single objective optimization approach to the energy allocation problem has received the attention of many researchers in the past (Sinha and Kandpal 1991a, 1991b,

1991c, 1992). There have been very few studies in the literature involving the use of operational research models for tackling such multi-objective energy resource allocation problems. In this paper, a multi-objective programming model with three objectives is discussed. The energy allocation process is viewed within a system framework by considering minimization of cost, maximization of use of locally available resources and minimization of use of external energy sources. The multi-objective allocation is carried out using goal programming to arrive at satisfactory solutions.

The model has been designed for energy allocation in the village Hosur, situated in Malnad region of Shimoga in Karnataka. The village is located at latitude 14° 30' north and longitude 75° 32' east, with a total population of 1850 in 300 households and 90% of the houses in that village are electrified. The geographic area of the village is 3562 acres, net cultivated land is 1659 acres, the remaining is the forest, waste land, non agricultural land etc. Mean average annual rainfall is 950 mm. Maize (92%) is the main crop grown in the village. Irrigation is available to only 5% of cropped area. The total livestock population is 470.

The major energy sources that are being used in Hosur village are electricity, firewood, agrowaste, kerosene etc. These energy sources are used for energy services like lighting and luxuries in domestic sector, water pumping for irrigation in agricultural sector and also for cooking, heating and transportation. The data were collected from 300 households of the village covering 100% of the families. Much of the data used for the analysis is based on the data obtained from detailed survey. The survey was conducted through structural written questioners containing primary and secondary data. The integrated energy plan for the selected village is prepared with 100% coverage basis, covering geographic and demographic characteristic of the village, economic activities and general living conditions of the people.

## II. OBJECTIVES

The present study has been carried out with the following objectives.

- To identify the local energy resources to meet the domestic energy needs for cooking, heating and for lighting purposes.

- To minimize the total energy cost of the village.
- To maximize the use of locally available energy resources and to minimize the use of external energy resources.
- To allocate the rural energy resources optimally.

### III. METHODOLOGY

• Data used for the analysis is obtained from detailed survey of various energy resources and demand of Hosur village, through structural written questionnaire. It also covers geographic and demographic characteristics of the village, economic activities and general living condition of the people.

- Energy consumption pattern of the village per month is tabulated.
- Three objective functions are taken into account for optimization of rural energy resources using goal programming.
- Demand and Supply Constraint equations are formulated.
- Linear programming model is prepared and feasibility is checked.
- The optimal values for the three objectives are obtained from linear programming model. Since it is a multi objective programming QM has been used for solving the goal programming.
- Results obtained by using goal programming model are compared with current energy consumption pattern of the village.

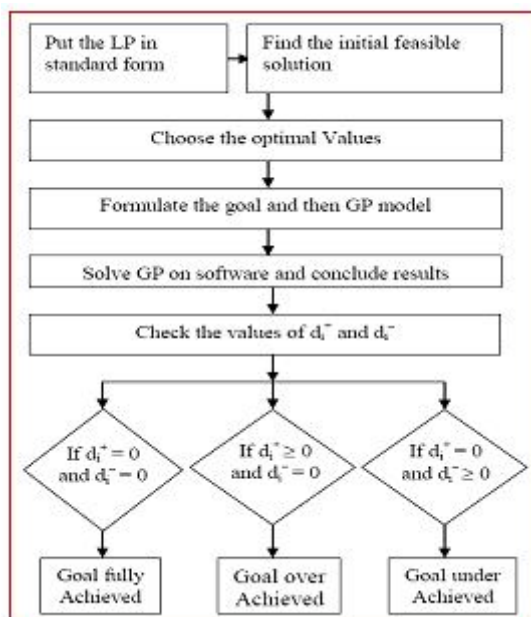


Figure1. Flow Chart

### IV. PROBLEM FORMULATION

Based on the data collected and discussion held with villagers, total energy consumption pattern were analysed. Table 1 gives the details of item wise energy consumption pattern and the cost of each energy sources. It is observed that the total energy consumption of Hosur village is 710441 MJ/month. The major contributors are fire wood and agro residue.

TABLE 1

ENERGY CONSUMPTION PATTERN OF HOSUR VILLAGE PER MONTH.

Sl. No.	Sources	Energy consumption per month	Cost in Rs	Unit Cost Rs/MJ
1	Electricity	21000 Kwh (75600)	3/Kwh	2
2	LPG	10 cylinders (8820)	30/kg	0.47
3	Kerosene	750 litres (28191)	15/litre	0.4
4	Petrol	250 litres (8280)	65/litre	1.96
5	Diesel	500 litres (19550)	44/litre	1.125
6	Firewood	25 tonnes (375000)	15/kg	1
7	Agrowaste	13 tonnes (195000)	5/kg	0.233
TOTAL		710441		

Figures in brackets gives energy equivalent in MJ per month of the village.

TABLE II. SOURCES AND USES OF ENERGY IN MJ AND %.

Usage→ Source↓	Cooking	Lighting	Heating	Transportation	Luxuries	Irrigation and cultivation
Electricity		9000 (1.26) [X <sub>2</sub> = 5]			16200 (2.28) [X <sub>12</sub> = 12]	50400 (7.09) [X <sub>13</sub> = 13]
LPG	8820 (1.24) [X <sub>1</sub> = 1]					
Kerosene	7831 (1.10) [X <sub>3</sub> = 2]	16800 (2.3) [X <sub>6</sub> = 6]	3417 (0.48) [X <sub>7</sub> = 7]			
Petrol				8280 (1.16) [X <sub>10</sub> = 10]		
Diesel				7678 (1.08) [X <sub>11</sub> = 11]		11872 (1.66) [X <sub>14</sub> = 14]
Fire wood	225085 (31.68) [X <sub>5</sub> = 3]		148915 (21.09) [X <sub>8</sub> = 8]			
Agro waste	116972 (16.46) [X <sub>4</sub> = 4]		78028 (10.9) [X <sub>9</sub> = 9]			

The problem is to calculate the optimal amount of energy to be supplied by a resource to a specific end-use i.e. the amount of energy to be allocated to a particular resource end- use combination. The resources, end-uses, and their combination have been considered on the basis of availability of sufficient data and the feasibility of resource utilization by households in Hosur village. In all, seven energy resources and six end-uses have been considered and for fourteen resources end-use combinations have been chosen. These are indicated in Table 2. It shows details regarding the type of energy used and the amount of energy used for end use services. Figures in brackets gives use of energy in % and figures in Square brackets show the energy resource end-use combinations which are numbered from 1 to 14.

#### A. Objective Functions

##### (i) Minimization of Cost:

The term cost refers to the actual cost of bringing the energy resource to the end-use point. This cost minimization

objective function is represented as:

$$\text{Min } \sum C_i x_i$$

i.e.,  $\text{Min } z1 = 2(x_{5+} x_{12+} x_{13}) + 0.47(x_1) + 0.4(x_2 + x_6 + x_7) + 1.96(x_{10}) + 1.125(x_{11} + x_{14}) + 1(x_3 + x_8) + 0.233(x_4 + x_9)$

Where  $C_i$  is the unit cost coefficient.

(ii) *Maximization of use of locally available resources*

The allocation process should prefer locally available resources to reduce the vulnerability of the use of other expensive energy sources. The objective function can be represented as:

$$\text{Max } \sum x_i \text{ Where } i = 3, 4, 8, 9$$

$$\text{i.e., Max } z2 = x_3 + x_4 + x_8 + x_9$$

(iii) *Minimization of use of external resources*

The demand and price of external energy resources like petrol and kerosene are increasing day by day, so their use should be minimized. The objective function can be represented as:

$$\text{Min } \sum C_i \text{ Where } i = 1, 2, 5, 6, 7, 10, 11, 12, 13, 14$$

$$\text{i.e., Min } z3 = x_1 + x_2 + x_5 + x_6 + x_7 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

### B. Constraints

#### a. Demand Constraints

- (i) Total cooking energy demand  $\geq 358707.1$  MJ/month  
i.e.,  $x_1 + x_2 + x_3 + x_4 \geq 358707.1$  MJ/month
- (ii) Total lighting demand  $\geq 25800$  MJ/month  
i.e.,  $x_5 + x_6 \geq 25800$  MJ/month
- (iii) Total heating demand  $\geq 231360.26$  MJ/month  
i.e.,  $x_7 + x_8 + x_9 \geq 231360.26$  MJ/month
- (iv) Total transportation demand  $\geq 5957.81$  MJ/month  
i.e.,  $x_{10} + x_{11} \geq 5957.81$  MJ/month
- (v) Total luxuries energy demand  $\geq 16200$  MJ/month  
i.e.,  $x_{12} \geq 16200$  MJ/month
- (vi) Total irrigation energy demand  $\geq 62272$  MJ/month  
i.e.,  $x_{13} + x_{14} \geq 62272$  MJ/month

#### b. Supply Constraints

- (i) Total electricity availability  $\leq 75600$  MJ/month  
i.e.,  $x_5 + x_{12} + x_{13} \leq 75600$  MJ/month
- (ii) Total LPG availability  $\leq 8820$  MJ/month  
i.e.,  $x_1 \leq 8820$  MJ/month
- (iii) Total kerosene availability  $\leq 28191$  MJ/month  
i.e.,  $x_2 + x_6 + x_7 \leq 28191$  MJ/month
- (iv) Total petrol availability  $\leq 8280$  MJ/month  
i.e.,  $x_{10} \leq 8280$  MJ/month
- (v) Total diesel availability  $\leq 19550$  MJ/month  
i.e.,  $x_{11} + x_{14} \leq 19550$  MJ/month
- (vi) Total firewood availability  $\leq 375000$  MJ/month  
i.e.,  $x_3 + x_8 \leq 375000$  MJ/month
- (vii) Total agrowaste availability  $\leq 195000$  MJ/month  
i.e.,  $x_4 + x_9 \leq 195000$  MJ/month

### C. Linear Programming Model (LP)

In order to meet energy planner's need for quantitative simulation a linear optimization model has been developed during the last several years. In this work, prior to the goal programming the feasibility of the formulated problem is checked using linear programming model. By considering the individual objective functions for all the combinations

separately, the feasible values for the minimization as well as the maximization of the objectives are calculated. These values are used for the formulation of the constraints for goal programming.

TABLE III. SUMMARY TABLE FOR LP SOLUTION

Variable	Status	Value
X1	Basic	8820
X2	NON Basic	0
X3	Basic	154887.1
X4	Basic	195000
X5	Basic	8856.391
X6	Basic	16943.61
X7	Basic	11247.39
X8	Basic	220112.4
X9	NON Basic	0
X10	Basic	8280
X11	Basic	7677.81
X12	Basic	16200
X13	Basic	50399.81
X14	Basic	11872.19
Surplus1	NON Basic	0
Surplus 2	NON Basic	0
Surplus 3	NON Basic	0
Surplus 4	NON Basic	0
Surplus 5	NON Basic	0
Surplus 6	NON Basic	0
Slack 7	Basic	143.7998
Slack 8	NON Basic	0
Slack 9	NON Basic	0
Slack 10	NON Basic	0
Slack 11	NON Basic	0
Slack 12	NON Basic	0
Slack 13	NON Basic	0
Optimal Value(Z1)		624991.8
Optimal Value(Z2)		570000
Optimal Value(Z3)		140297.2

Table 3 shows the summary of the results obtained by linear programming. The three objective functions taken have been solved separately considering the same constraints using the QM (Quantitative Method) for windows software. The optimal values of Z1, Z2 and Z3 have been tabulated. These values are used during the formulation of the Goal Programming.

### D. Goal Programming Model (GP)

In a Goal Programming problem, there are multiple objectives and the deviations from constraints are penalized. The optimization model used in the study consists of 3 objective functions. This model cannot be solved by using ordinary linear optimization and hence goal programming has been employed to solve the optimization problem. These 3 goals may be either *over* or *under* achieved. Deviation variables are introduced to represent the over or under achievement of the goals:

d1+ represents the amount of over-achievement of goal (1)

d1- represents the amount of under-achievement of goal (1)

d2+ represents the amount of over-achievement of goal (2)

d2- represents the amount of under-achievement of goal (2)

d3+ represents the amount of over-achievement of goal (3)

d3- represents the amount of under-achievement of goal (3)

Note that for any goal K:

If the goal is exactly achieved:

$$d_1^+ = 0, d_1^- = 0$$

If the goal is over-achieved:

$$d_1^+ > 0, d_1^- = 0$$

If the goal is under-achieved:

$$d_1^+ = 0, d_1^- > 0$$

The objective of the goal programming is to minimize deviations from the goals given by:

Minimize: " $d_1^- + d_1^+$  where  $i = 1, 2, 3$

i.e., Minimize  $(d_1^- + d_1^+ + d_2^- + d_2^+ + d_3^- + d_3^+)$

Adding pairs of deviation variables to the goals (all three goals separately) transforms them into a set of constraints:

Subjected to  $L_i + w_i d_i^- - w_i d_i^+ = b_i$

Where  $L_i = i^{\text{th}}$  Objective function

$w_i = b_i - f_i =$  weighing factor for  $d_i^-$  and  $d_i^+$

$b_i =$  goal for the objective  $i$

$f_i =$  worst possible value for objective  $i$

$=$  minimum value for objective  $i$

(for maximization type problem)

$=$  maximum value for objective  $i$

(for minimization type problem)

i.e.,

$$\begin{aligned} & \geq 2(x_5 + x_{12} + x_{13}) + 0.47(x_1) + 0.4(x_2 + x_6 + x_7) + \\ & 1.96(x_{10}) + 1.125(x_{11} + x_{14}) + 1(x_3 + x_8) + 0.233(x_4 + x_9) + w_1 d_1^- - \\ & w_1 d_1^+ = b_1 = 624991.8 \end{aligned}$$

..... (1)

It can be written as :

$$\begin{aligned} & \geq 2(x_5 + x_{12} + x_{13}) + 0.47(x_1) + 0.4(x_2 + x_6 + x_7) + 1.96(x_{10}) + \\ & 1.125(x_{11} + x_{14}) + 1(x_3 + x_8) + 0.233(x_4 + x_9) + 241135.4 d_1^- = \\ & 624991.8 \end{aligned}$$

$$\geq x_3 + x_4 + x_8 + x_9 + w_2 d_2^- - w_2 d_2^+ = b_2 = 570000 \quad \dots (2)$$

It can be written as :

$$\geq x_3 + x_4 + x_8 + x_9 + 561863.8 d_1^- = 570000$$

$$\geq x_1 + x_2 + x_5 + x_6 + x_7 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + w_3 d_3^- - w_3 d_3^+ = b_3 = 140297.2 \quad \dots (3)$$

It can be written as:

$$\geq x_1 + x_2 + x_5 + x_6 + x_7 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + 243559.2 d_1^+ = 140297.2$$

Where,

$d_1^- = d_1^+ = 1$  (due to priority of the objective (i) being 1)

$d_2^- = d_2^+ = 2$  (due to priority of the objective (ii) being 2)

$d_3^- = d_3^+ = 3$  (due to priority of the objective (iii) being 3)

$b_j =$  goal for objective  $j$  and  $w_j =$  weighing factor for goal  $j$

The other constraints are:

(i) Total cooking energy demand  $\geq 358707.1$  MJ/month  
i.e.,  $x_1 + x_2 + x_3 + x_4 \geq 358707.1$  MJ/month

(ii) Total lighting demand  $\geq 25800$  MJ/month  
i.e.,  $x_5 + x_6 \geq 25800$  MJ/month

(iii) Total heating demand  $\geq 231360.26$  MJ/month  
i.e.,  $x_7 + x_8 + x_9 \geq 231360.26$  MJ/month

(iv) Total transportation demand  $\geq 15957.81$  MJ/month

i.e.,  $x_{10} + x_{11} \geq 15957.81$  MJ/month

(v) Total luxuries energy demand  $\geq 16200$  MJ/month

i.e.,  $x_{12} \geq 16200$  MJ/month

(vi) Total irrigation energy demand  $\geq 62272$  MJ/month

i.e.,  $x_{13} + x_{14} \geq 62272$  MJ/month

(vii) Total electricity availability  $\leq 75600$  MJ/month

i.e.,  $x_5 + x_{12} + x_{13} \leq 75600$  MJ/month

(viii) Total LPG availability  $\leq 8820$  MJ/month

i.e.,  $x_1 \leq 8820$  MJ/month

(ix) Total kerosene availability  $\leq 28191$  MJ/month

i.e.,  $x_2 + x_6 + x_7 \leq 28191$  MJ/month

(x) Total petrol availability  $\leq 8280$  MJ/month

i.e.,  $x_{10} \leq 8280$  MJ/month

(xi) Total diesel availability  $\leq 19550$  MJ/month

i.e.,  $x_{11} + x_{14} \leq 19550$  MJ/month

(xii) Total firewood availability  $\leq 375000$  MJ/month

i.e.,  $x_3 + x_8 \leq 375000$  MJ/month

(xiii) Total agro waste availability  $\leq 195000$  MJ/month

i.e.,  $x_4 + x_9 \leq 195000$  MJ/month

The following procedure adopted to fix the goals and worst values for the three objectives. First each of the three objectives is individually optimised and the optimised value for each objective is fixed as the corresponding goal then the three optimal solutions thus obtained are employed in each objective functions and the minimum value (for maximization objectives) or maximum value (for minimization objectives) has been taken as the value of  $f_i$  to be used for the corresponding objective. The value of  $b_j$  and  $f_j$  and  $w_j$  are shown in the table 4.

TABLE IV . CALCULATION OF WEIGHING FACTOR

Objective number j	Goal $b_j$	worst value $f_j$	Weighing factor $w_j = b_j - f_j$
1	624991.8	383856.4	241135.4
2	570000	8136.2	561863.8
3	140297.2	383856.4	-243559.2

## V. RESULTS AND DISCUSSION

Table 5 shows that priority wise non achievement is - 239.05, -1.01 and 35023050 respectively. If there are no positive values, there is no non achievement in a priority. In the results first and second priorities are negative which indicates that first priority goal i.e., minimization of cost and second priority goal i.e., maximization of use of locally available energy resources is prioritised more effectively. Priority third value is positive which means that the priority of third goal has been under achieved.

For goal 1  $d_1^+ > 0$  and  $d_1^- = 0$ , so objective of minimization of cost is over achieved.

For goal 2  $d_2^+ = 0$  and  $d_2^- = 0$ , so objective of maximization of use of locally available resources is fully achieved.

For goal 3  $d_3^+ > 0$  and  $d_3^- = 0$ , so objective of minimization of use of external resources is also over achieved.



TABLE V. PRIORITY AND GOAL RESULT OBTAINED BY SOFTWARE SOLUTION

Priority analysis	Non achievement		
Priority 1	-239.05		
Priority 2	-1.01		
Priority 3	35023050		
Constraint analysis	RHS	d <sup>+</sup> (row i)	d <sup>-</sup> (row i)
Goal /constraint 1	624991.8	287.5	0
Goal /constraint 2	570000	0	0
Goal /constraint 3	140297.2	143.8	0
Goal /constraint 4	358707.1	143.81	0
Goal /constraint 5	25800	0	0
Goal /constraint 6	231360.3	0	0
Goal /constraint 7	15957.81	0	0
Goal /constraint 8	16200	0	0
Goal /constraint 9	62272	0	0
Goal /constraint 10	75600	0	0
Goal /constraint 11	8820	0	0
Goal /constraint 12	28191	0	0
Goal /constraint 13	8280	0	0
Goal /constraint 14	19550	0	0
Goal /constraint 15	375000	0	0
Goal /constraint 16	195000	0	0

TABLE VI. ENERGY DATA TABLE AFTER GOAL PROGRAMMING

Usage → Source ↓	Cooking	Lighting	Heating	Transportation	Luxuries	Irrigation and cultivation
Electricity		9000.19			1620 0	50399.8 1
LPG	8820					
Kerosene	0	16799.8 1	11391.1 9			
Petrol				8280		
Diesel				7677.8		11872.1 9
Fire wood	350030. 9		24969.1 1			
Agro waste	0		195000			

## VI. CONCLUSION

The model prescribes different energy resources that are competitive (in the sense specified by the three objectives stated earlier) in meeting specific end-uses. Under no externally imposed availability constraints, optimal allocation favours the use of agro waste and firewood. The result broadly shows a preference of agro waste and firewood. Thus this normative study leads to the following conclusions.

- The use of agro waste for heating should be increased from 78028 MJ to 195000 MJ.
- The use of firewood for cooking should be increased from 225085 MJ to 350030.9 MJ.
- The use of Kerosene should be limited to 11391.19 MJ for heating and its use should be avoided for cooking.
- For heating purpose 24969.11 MJ of fire wood energy should use instead of 149914.74 MJ.
- To meet the demand of cooking, firewood should be promoted and the use of agro waste and Kerosene should be avoided.

- To meet the demand of heating, Kerosene and Agro waste should be promoted.

Although the above conclusions are made with special references to the village Hosur of malnad region, they are, in general, applicable to any village or urban environment.

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